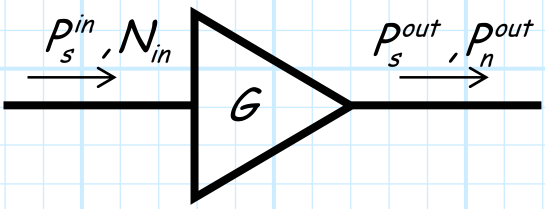
Noise Figure and SNR

Of course, in addition to noise, the input to an amplifier in a receiver will typically include our desired **signal**.

Say the **power** of this input signal is P_s^{in}. The output of the amplifier will therefore include **both** a signal with power P_s^{out}, and noise with power P_n^{out}:



where:

$$P_s^{out} = G\,P_s^{in}$$

and:

$$\begin{aligned} P_n^{out} &= N_{in} + G\,k\,T_e\,B \\ &= G\,k\left(T_{in} + T_e\right)B \end{aligned}$$

In order to accurately demodulate the signal, it is important that signal power be **large** in comparison to the noise power. Thus, a fundamental and important measure in radio systems is the **Signal-to-Noise Ratio** (SNR):

$$SNR \doteq \frac{P_s}{P_n}$$

The **larger** the SNR, the **better**!

At the **output** of the amplifier, the SNR is:

$$\begin{aligned} SNR_{out} &= \frac{P_s^{out}}{P_n^{out}} \\ &= \frac{G\,P_s^{in}}{G\,k\left(T_{in}+T_e\right)B} \\ &= \frac{P_s^{in}}{k\left(T_{in}+T_e\right)B} \end{aligned}$$

Moreover, we can define an **input noise power** as the total noise power across the **bandwidth of the amplifier**:

$$P_n^{in} = N_{in}\,B = k\,T_{in}\,B$$

And thus the **input** SNR as:

$$SNR_{in} = \frac{P_s^{in}}{P_n^{in}} = \frac{P_s^{in}}{k\,T_{in}B}$$

Now, let's take the **ratio** of the input SNR to the output SNR:

$$\begin{aligned} \frac{SNR_{in}}{SNR_{out}} &= \frac{P_s^{in}}{k\,T_{in}B}\left(\frac{k\left(T_{in}+T_e\right)B}{P_s^{in}}\right) \\ &= \frac{T_{in}+T_e}{T_{in}} \\ &= 1+\frac{T_e}{T_{in}} \end{aligned}$$

Since $T_e > 0$, it is evident that:

$$\frac{SNR_{in}}{SNR_{out}} = 1 + \frac{T_e}{T_{in}} > 1$$

In other words, the SNR at the **output** of the amplifier will be **less** than the SNR at the **input**.

→ This is very **bad** news!

This result means that the SNR will always be **degraded** as the signal passes through **any** microwave component!

As a result, the SNR at the **input** of a receiver will be the largest value it will **ever** be within the receiver. As the signal passes through each component of the receiver, the SNR will get steadily **worse**!

Q: *Why is that? After all, if we have several amplifiers in our receiver, the **signal** power will significantly **increase**?*

A: True! But remember, this gain will likewise increase the receiver input **noise** by the **same** amount. Moreover, each component will add **even more noise**—the internal noise produced by each receiver component.

Thus, the power of a signal traveling through a receiver increases—but the **noise** power increases **even more**!

Note that the ratio SNR_{in}/SNR_{out} essentially quantifies the degradation of SNR by an amplifier—a ratio of **one** is **ideal**, a **large** ratio is very **bad**.

So, let's go back and look **again** at ratio SNR_{in}/SNR_{out}:

$$\frac{SNR_{in}}{SNR_{out}} = 1 + \frac{T_e}{T_{in}}$$

Note what this ratio **depends** on, and what it does **not**.

This ratio **depends** on:

1. T_e (a device parameter)

2. T_{in} (**not** a device parameter)

This ratio does not **depend** on:

1. The amplifier gain G.

2. The amplifier bandwidth B.

We thus might be tempted to use the ratio SNR_{in}/SNR_{out} as another **device parameter** for describing the **noise** performance of an amplifier. After all, SNR_{in}/SNR_{out} depends on T_e, but does **not** depend on other device parameters such as G or B.

Moreover, SNR is a value that can generally be easily **measured**!

But the problem is the **input** noise temperature T_{in}. This can be **any** value—it is **independent** of the amplifier itself.

For **example**, it is event that as the input noise increases to **infinity**:

$$\lim_{T_{in}\to\infty} \frac{SNR_{in}}{SNR_{out}} = \lim_{T_{in}\to\infty}\left(1+\frac{T_e}{T_{in}}\right) = 1$$

In other words, if the input noise is large enough, the internally generated amplifier noise will become **insignificant**, and thus will degrade the SNR **very little**!

Q: *Degrade the SNR very little! This means $SNR_{out} = SNR_{in}$! Isn't this* ***desirable****?*

A: Not in this instance. Note that if T_{in} increases to infinity, then:

$$\lim_{T_{in}\to\infty} SNR_{in} = \lim_{T_{in}\to\infty}\left(\frac{P_s^{in}}{kT_{in}B}\right) = 0$$

In other words, the SNR does is not degraded by the amplifier **only** because the SNR is already as bad (i.e., $SNR = 0$) as it can possibly get!

Conversely, as the input noise temperature decreases toward **zero**, we find:

$$\lim_{T_{in}\to 0} \frac{SNR_{in}}{SNR_{out}} = \lim_{T_{in}\to 0} \left(1 + \frac{T_e}{T_{in}}\right) = \infty$$

Q: *Yikes! The amplifier degrades the SNR by an infinite percentage! Isn't this **undesirable**?*

A: Not in this instance. Note that if T_{in} decreases to zero, then:

$$\lim_{T_{in}\to 0} SNR_{in} = \lim_{T_{in}\to 0} \left(\frac{P_s^{in}}{k\,T_{in}\,B}\right) = \infty$$

Note this is the **perfect** SNR, and thus the ratio SNR_{in}/SNR_{out} will likewise be infinity, **regardless** of the amplifier.

Anyway, the **point** here is that although the degradation of SNR by the amplifier does depend on the **amplifier** noise characteristics (i.e., T_e), it **also** on the noise input to the amplifier (i.e., T_{in}).

This input noise is a variable that is unrelated to amplifier performace

Q: *So there is **no way** to use SNR_{in}/SNR_{out} as a **device parameter?***

A: Actually there **is**! In fact, it is the most **prevalent** parameter for specifying microwave device noise performance. This measure is called **noise figure.**

The noise figure of a device **is** simply the measured ratio SNR_{in}/SNR_{out} exhibited by a device, **for a specific input noise temperature** T_{in}.

I **repeat**:

→ "**for a specific input noise temperature** T_{in}."

This specific noise temperature is almost **always** taken as the standard "room temperature" of $T_o = 290\,K^\circ$. Note this was likewise the standard **antenna noise temperature** assumption.

Thus, the **Noise Figure** (F) of a device is defined as:

$$F \doteq \left.\frac{SNR_{in}}{SNR_{out}}\right|_{T_{in}=290K^\circ}$$

$$= \left.\left(1 + \frac{T_e}{T_{in}}\right)\right|_{T_{in}=290K^\circ}$$

$$= 1 + \frac{T_e}{290K^\circ}$$

It is critically important that **you** understand the **definition** of noise figure. A common **mistake** is to assume that:

$$SNR_{out} = \frac{SNR_{in}}{F}$$ ← This is **not** generally true!

Note this would only be true **if** $T_{in} = 290K^{\circ}$, but this is almost **never** the case (i.e., $T_{in} \neq 290K^{\circ}$ generally speaking).

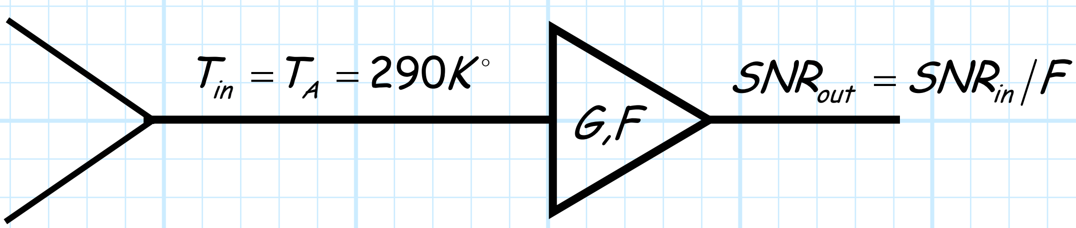
Thus, an **incorrect** (but **widely repeated**) statement would be:

" The noise figure specifies the degradation of SNR."

Whereas, a **correct** statement is:

*" The noise figure specifies the degradation of SNR, for the specific condition when $T_{in} = 290K^{\circ}$, and for that specific condition **only**"*

The one **exception** to this is when an **antenna** is connected to the input of an amplifier. For this case, it is evident that the input temperature **is** $T_A = T_{in} = 290K^{\circ}$:



Note that since the noise figure F of a given device is dependent on its equivalent noise temperature T_e, we can **determine** the equivalent noise temperature T_e of a device with knowledge F:

$$F = 1 + \frac{T_e}{290K^\circ} \qquad \Leftrightarrow \qquad T_e = (F-1)290K^\circ$$

One **more** point. Note that noise figure F is a **unitless** value (just like gain!). As such, we can easily express it in terms of **decibels** (just like gain!):

$$F(dB) = 10\, log_{10} F$$

Like gain, the noise figure of an amplifier is **typically** expressed in dB.